Oefening 11

$$\begin{aligned} x &= e^t \cos t \\ y &= e^t \sin t \\ z &= e^t \end{aligned}$$

- $\mathbf{1_t} = \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$

$$\begin{aligned} \dot{x} &= e^t (\cos t - \sin t) \\ \dot{y} &= e^t (\sin t + \cos t) \\ \dot{z} &= e^t \\ &\Downarrow \\ \dot{\mathbf{r}} &= (e^t (\cos t - \sin t), e^t (\sin t + \cos t), e^t) \\ &\Downarrow \\ |\dot{\mathbf{r}}| &= e^t \sqrt{1 - 2\cos t \sin t + 1 + 2\cos t \sin t + 1} \\ &= \sqrt{3} e^t \\ &\Downarrow \end{aligned}$$

$$\mathbf{1_t} = \frac{1}{\sqrt{3}}(\cos t - \sin t, \sin t + \cos t, 1)$$

- $\mathbf{1_n}$

  Frenet:

$$\frac{d\mathbf{1_t}}{ds} = \frac{1}{R}\mathbf{1_n}$$

$$\Downarrow$$

$$\mathbf{1_n} = \frac{\frac{d\mathbf{1_t}}{ds}}{\left|\frac{d\mathbf{1_t}}{ds}\right|} = \frac{\frac{d\mathbf{1_t}}{dt}\frac{dt}{ds}}{\left|\frac{d\mathbf{1_t}}{dt}\frac{dt}{ds}\right|}$$

$$\Downarrow \quad \frac{dt}{ds} = \frac{1}{\frac{ds}{dt}} = \frac{1}{|\dot{\mathbf{r}}|} > 0$$

$$\mathbf{1_n} = \frac{\frac{d\mathbf{1_t}}{dt}\frac{dt}{ds}}{\left|\frac{d\mathbf{1_t}}{dt}\right|\frac{dt}{ds}} = \frac{\frac{d\mathbf{1_t}}{dt}}{\left|\frac{d\mathbf{1_t}}{dt}\right|}$$

$$\frac{d\mathbf{1_t}}{dt} = \frac{1}{\sqrt{3}}(-\sin t - \cos t, \cos t - \sin t, 0)$$

$$\Downarrow$$

$$\begin{aligned}\left|\frac{d\mathbf{1_t}}{dt}\right| &= \frac{1}{\sqrt{3}}\sqrt{1+2\sin t\cos t+1-2\sin t\cos t} \\ &= \frac{\sqrt{2}}{\sqrt{3}} \\ &\Downarrow \\ \mathbf{1_n} &= \frac{1}{\sqrt{2}}(-\sin t-\cos t,\cos t-\sin t,0)\end{aligned}$$

- $\mathbf{1_b}$

$$\begin{aligned}\mathbf{1_b} &= \mathbf{1_t}\times\mathbf{1_n} \\ &= \frac{1}{\sqrt{6}}\begin{vmatrix}\mathbf{1_x} & \mathbf{1_y} & \mathbf{1_z} \\ \cos t-\sin t & \sin t+\cos t & 1 \\ -\sin t-\cos t & \cos t-\sin t & 0\end{vmatrix} \\ &= \frac{1}{\sqrt{6}}(\sin t-\cos t,-\sin t-\cos t,2)\end{aligned}$$

- Kromming

$$k = \frac{1}{R}$$

$$\Downarrow \quad \text{Frenet} : \frac{d\mathbf{1_t}}{ds} = \frac{1}{R}\mathbf{1_n}$$

$$k = \left|\frac{d\mathbf{1_t}}{ds}\right| = \left|\frac{d\mathbf{1_t}}{dt}\frac{dt}{ds}\right|$$

$$\Downarrow \quad \frac{dt}{ds} = \frac{1}{\frac{ds}{dt}} = \frac{1}{|\dot{\mathbf{r}}|} = \frac{1}{\sqrt{3}e^t}$$

$$k = \frac{1}{\sqrt{3}e^t}\left|\frac{d\mathbf{1_t}}{dt}\right| = \frac{\sqrt{2}}{3e^t}$$

- Torsie

  Frenet:

$$\frac{d\mathbf{1_b}}{ds} = \frac{1}{T}\mathbf{1_n}$$

$$\Downarrow$$

$$\frac{1}{T} = \left|\frac{d\mathbf{1_b}}{ds}\right| = \left|\frac{d\mathbf{1_b}}{dt}\frac{dt}{ds}\right|$$

$$\Downarrow \quad \frac{dt}{ds} = \frac{1}{\frac{ds}{dt}} = \frac{1}{|\dot{\mathbf{r}}|} = \frac{1}{\sqrt{3}e^t}$$

$$\frac{1}{T} = \frac{1}{\sqrt{3}e^t}\left|\frac{d\mathbf{1_b}}{dt}\right|$$

$$\begin{aligned}
\frac{d\mathbf{1_b}}{dt} &= \frac{1}{\sqrt{6}}(\sin t+\cos t,\sin t-\cos t,0)\\
&\Downarrow\\
\left|\frac{d\mathbf{1_b}}{dt}\right| &= \frac{1}{\sqrt{6}}\sqrt{1+2\sin t\cos t+1-2\sin t\cos t}\\
&= \frac{\sqrt{2}}{\sqrt{6}}=\frac{1}{\sqrt{3}}\\
&\Downarrow\\
\frac{1}{T} &= \frac{1}{3e^t}
\end{aligned}$$

- Raaklijn

$$\begin{gathered}
\tfrac{x-e^t\cos t}{e^t(\cos t-\sin t)}=\tfrac{y-e^t\sin t}{e^t(\sin t+\cos t)}=\tfrac{z-e^t}{e^t}\\
\Downarrow\\
\tfrac{x-e^t\cos t}{\cos t-\sin t}=\tfrac{y-e^t\sin t}{\sin t+\cos t}=z-e^t
\end{gathered}$$

- Normaalvlak

$$
\begin{gathered}
e^t(\cos t-\sin t)(x-e^t\cos t) \\
+e^t(\sin t+\cos t)(y-e^t\sin t)+e^t(z-e^t)=0 \\
\Downarrow \\
(\cos t-\sin t)(x-e^t\cos t) \\
+(\sin t+\cos t)(y-e^t\sin t)+z-e^t=0
\end{gathered}
$$

- Osculatievlak

$$
e^{2t}\begin{vmatrix} x-e^t\cos t & y-e^t\sin t & z-e^t \\ \cos t-\sin t & \sin t+\cos t & 1 \\ -2\sin t & 2\cos t & 1 \end{vmatrix}=0
$$

$$\Downarrow$$

$$
\begin{gathered}
(x-e^t\cos t)(\sin t+\cos t-2\cos t) \\
-(y-e^t\sin t)(\cos t-\sin t+2\sin t) \\
+(z-e^t)[2\cos t(\cos t-\sin t) \\
+2\sin t(\sin t+\cos t)]=0 \\
\Downarrow
\end{gathered}
$$

$$(x - e^t \cos t)(\sin t - \cos t)$$
$$-(y - e^t \sin t)(\cos t + \sin t) + 2(z - e^t) = 0$$