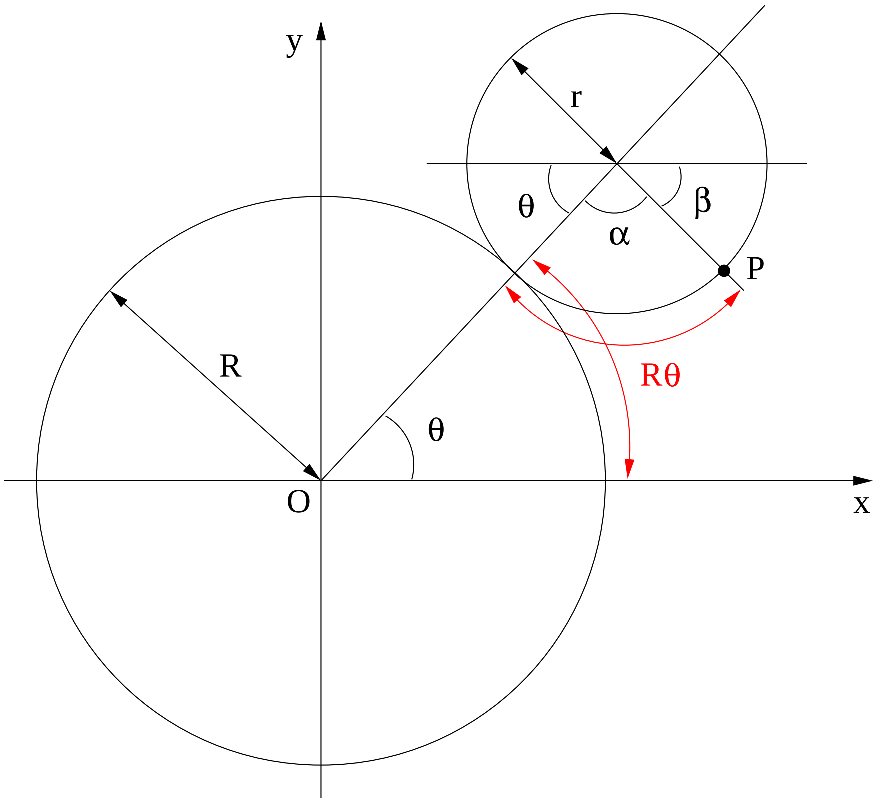
Oefening 20 (epicycloïde)

- Parametervergelijkingen:



$$\left\{ \begin{array}{lll} x & = & (R+r)\cos\theta + r\cos\beta \\ y & = & (R+r)\sin\theta - r\sin\beta \end{array} \right.$$

$$R\theta \quad = \quad r\alpha \Rightarrow \alpha = \frac{R}{r}\theta$$

$$\pi = \theta + \alpha + \beta \Rightarrow \beta = \pi - \theta\left(1 + \frac{R}{r}\right)$$

$$\Rightarrow \begin{cases} x = (R+r)\cos\theta + r\cos\left[\pi - \theta\left(1+\frac{R}{r}\right)\right] \\ y = (R+r)\sin\theta - r\sin\left[\pi - \theta\left(1+\frac{R}{r}\right)\right] \end{cases}$$

$$\Rightarrow \begin{cases} x = (R+r)\cos\theta - r\cos\left[\theta\left(1+\frac{R}{r}\right)\right] \\ y = (R+r)\sin\theta - r\sin\left[\theta\left(1+\frac{R}{r}\right)\right] \end{cases}$$

- Kromming:

$$\mathcal{R} = \pm\frac{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}}$$

$$\dot{x} = -(R+r)\sin\theta + (R+r)\sin\left[\theta\left(1+\frac{R}{r}\right)\right]$$

$$\dot{y} = (R+r)\cos\theta - (R+r)\cos\left[\theta\left(1+\frac{R}{r}\right)\right]$$

$$\Downarrow$$

$$\dot{x}^2 + \dot{y}^2 = (R+r)^2\left(\sin^2\theta - 2\sin\theta\sin\left[\theta\left(1+\frac{R}{r}\right)\right] + \sin^2\left[\theta\left(1+\frac{R}{r}\right)\right] + \cos^2\theta\right.$$

$$
\begin{aligned}
& -2\cos\theta\cos\left[\theta\left(1+\frac{R}{r}\right)\right] \\
& +\cos^2\left[\theta\left(1+\frac{R}{r}\right)\right]\Bigg) \\
= \quad & 2\left(R+r\right)^2\left(1-\sin\theta\sin\left[\theta\left(1+\frac{R}{r}\right)\right]\right. \\
& \left.-\cos\theta\cos\left[\theta\left(1+\frac{R}{r}\right)\right]\right) \\
\Downarrow \quad & \cos\left(\alpha-\beta\right)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \\
\dot{x}^2+\dot{y}^2 \quad = \quad & 2\left(R+r\right)^2\left(1-\cos\left(\theta\frac{R}{r}\right)\right) \\
\Downarrow \quad & 1-\cos\alpha=2\sin^2\frac{\alpha}{2} \\
\dot{x}^2+\dot{y}^2 \quad = \quad & 4\left(R+r\right)^2\sin^2\left(\frac{\theta R}{2r}\right)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{x} &= \left(R+r\right)\left(-\cos\theta+\left(1+\frac{R}{r}\right)\cos\left[\theta\left(1+\frac{R}{r}\right)\right]\right) \\
\ddot{y} &= \left(R+r\right)\left(-\sin\theta+\left(1+\frac{R}{r}\right)\sin\left[\theta\left(1+\frac{R}{r}\right)\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}\ddot{y}-\ddot{x}\dot{y} &= (R+r)^2\left[2+\frac{R}{r}\right.\\
&\quad -\left(2+\frac{R}{r}\right)\left(\sin\theta\sin\left[\theta\left(1+\frac{R}{r}\right)\right]\right.\\
&\quad \left.\left.+\cos\theta\cos\left[\theta\left(1+\frac{R}{r}\right)\right]\right)\right]\\
&= (R+r)^2\left(2+\frac{R}{r}\right)\left(1-\cos\left(\theta\frac{R}{r}\right)\right)\\
&= 2(R+r)^2\left(2+\frac{R}{r}\right)\sin^2\left(\frac{\theta R}{2r}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \mathcal{R} &= \frac{8(R+r)^3\sin^3\left(\frac{\theta R}{2r}\right)}{2(R+r)^2\left(2+\frac{R}{r}\right)\sin^2\left(\frac{\theta R}{2r}\right)}\\
&= \frac{4r(R+r)}{2r+R}\sin\left(\frac{\theta R}{2r}\right)\\
&\Downarrow\\
k=\frac{1}{\mathcal{R}} &= \frac{2r+R}{4r(R+r)}\sin^{-1}\left(\frac{\theta R}{2r}\right)
\end{aligned}
$$